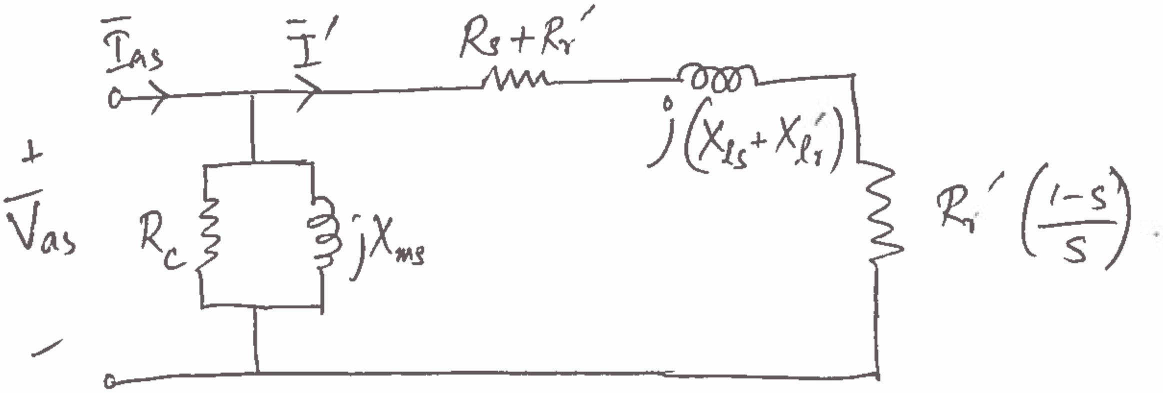
Torque-speed or Torque-slip characteristics.

Recall the approximate equivalent circuit for an induction machine.



P_{mech} = mechanical power developed

= power consumed in $R_r'\left(\frac{1-s}{s}\right)$.

$= 3 \cdot |\bar{I}'|^2 \cdot R_r'\left(\frac{1-s}{s}\right)$

why 3?

Reason it for yourself!

$$T^e = \frac{P_{mech}}{\omega_m} = \frac{P_{mech}}{\frac{2}{p}\cdot(1-s)\cdot\omega_s}$$

$$= \frac{3|\bar{I}'|^2 \cdot R_r'\left(\frac{\cancel{1-s}}{s}\right)}{\frac{2}{p}\cdot(\cancel{1-s})\cdot\omega_s}$$

What is $\bar{I}'$?

$$\bar{I}' = \frac{\bar{V}_{as}}{\underbrace{R_s + R_r' + R_r'\left(\frac{1-s}{s}\right)}_{R_s + R_r'/s} + j\underbrace{\left(X_{ls} + X_{lr}'\right)}_{= X}}$$

$$\therefore \bar{I}' = \frac{\bar{V}_{as}}{R_s + \frac{R_r'}{s} + jX}.$$

$$\Rightarrow T^e = 3 \cdot \left[\frac{|\bar{V}_{as}|^2}{\left(R_s + \frac{R_r'}{s}\right)^2 + X^2} \right] \cdot \frac{R_r'}{s} \cdot \frac{1}{\frac{2}{p} \cdot \omega_s}.$$

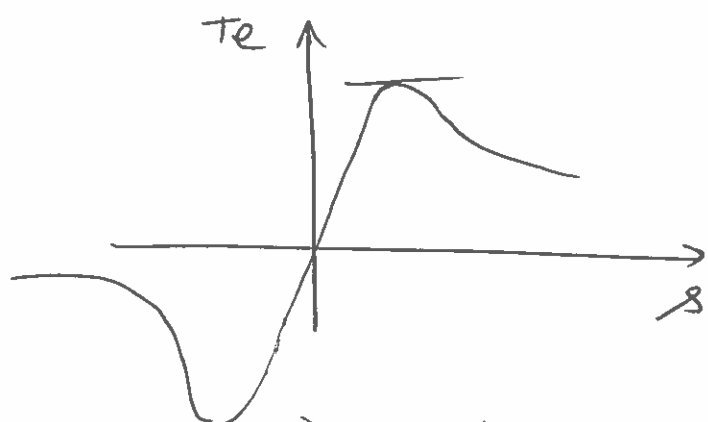
$$= \frac{3p|\bar{V}_{as}|^2}{2\omega_s} \cdot R_r' \cdot \frac{1}{s\left[\left(R_s + \frac{R_r'}{s}\right)^2 + X^2\right]}.$$

Two salient features are of interest :-

① How T^e behaves with s.

— See graphs attached herewith.

It looks somewhat like this....



② Find $\max\limits_{s} |T^e(s)|$, and $s^* = \arg\max\limits_{s} |T^e(s)|$.

$$T^e(s) = \underbrace{\left(\frac{3 \cdot p \cdot |\bar{V}_{as}|^2}{2\,\omega_s}\right)}_{:= \eta} \cdot R_r' \cdot \frac{1}{s\left[\left(R_s + \frac{R_r'}{s}\right)^2 + X^2\right]}.$$

η ← some constant.

$$\frac{\partial T^e(s)}{\partial s} = \eta \cdot R_r' \cdot \frac{s\left[2\left(R_s + \frac{R_r'}{s}\right)\left(\frac{-R_r'}{s^2}\right)\right] + \left[\left(R_s + \frac{R_r'}{s}\right)^2 + X^2\right]}{s^2\left[\left(R_s + \frac{R_r'}{s}\right)^2 + X^2\right]^2}$$

Set $\left.\frac{\partial T^e(s)}{\partial s}\right|_{s^*} = 0$ ...... Solving for s^* does not involve the denominator in $\frac{\partial T^e}{\partial s}$.

$$\Rightarrow \cancel{s^*}\left[2\left(R_s + \frac{R_r'}{s^*}\right)\cdot\left(-\frac{R_r'}{\cancel{s^{*2}}}\right)\right] + \left(R_s + \frac{R_r'}{s^*}\right)^2 + X^2 = 0.$$

$$\Rightarrow \left(R_s + \frac{R_r'}{s^*}\right)\left[-\frac{2R_r'}{s^*} + R_s + \frac{R_r'}{s^*}\right] + X^2 = 0$$

$$\Rightarrow \left(R_s + \frac{R_r'}{s^*}\right)\left(R_s - \frac{R_r'}{s^*}\right) + X^2 = 0$$

$$\Rightarrow R_s^2 - \frac{(R_r')^2}{s^{*2}} + X^2 = 0.$$

$$\Rightarrow \quad \frac{(R_r')^2}{s^{*2}} = R_s^2 + X^2 .$$

$$\Rightarrow \quad \boxed{s^* = \pm \frac{R_r'}{\sqrt{R_s^2 + X^2}}}$$

Let's evaluate $T^e(s)$ at both candidate values for s^* we obtained above.

$$T^e(s^*) = \eta \cdot \cancel{R_r'} \frac{1}{\left(\pm \frac{\cancel{R_r'}}{\sqrt{R_s^2+X^2}}\right)\left[\left(R_s \pm \frac{\cancel{R_r'}\sqrt{R_s^2+X^2}}{\cancel{R_r'}}\right)^2 + X^2\right]}$$

$$= \eta \cdot \cancel{R_r'} \cdot \frac{\sqrt{R_s^2+X^2}}{\left(R_s \pm \sqrt{R_s^2+X^2}\right)^2 + X^2}$$

$$= \pm \eta \cdot \frac{\sqrt{R_s^2+X^2}}{R_s^2 + (R_s^2+X^2) \pm 2R_s\sqrt{R_s^2+X^2} + X^2}$$

$$= \pm \eta \cdot \frac{\cancel{\sqrt{R_s^2+X^2}}}{\cancel{\sqrt{R_s^2+X^2}}\left[2\sqrt{R_s^2+X^2} + 2R_s\right]} .$$

$$= \pm \frac{\eta}{2} \cdot \frac{1}{R_s \mp \sqrt{R_s^2 + X^2}}.$$

$\therefore$ $|T^e(s)|$ is maximized at both candidate solutions s^*.

$$\max_s |T^e(s)| = \underbrace{\frac{\eta}{2} \cdot \frac{1}{R_s + \sqrt{R_s^2 + X^2}}.}$$

Play around with the matlab code to see it.

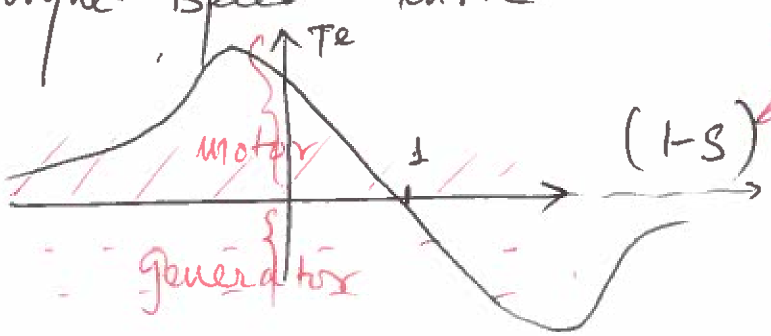
Observation: Does not depend on R_r' !!.

Two last things to know:

1. $T^e > 0 \Rightarrow$ induction machine is a motor

   $T^e < 0 \Rightarrow$ - - - - - - - - - generator

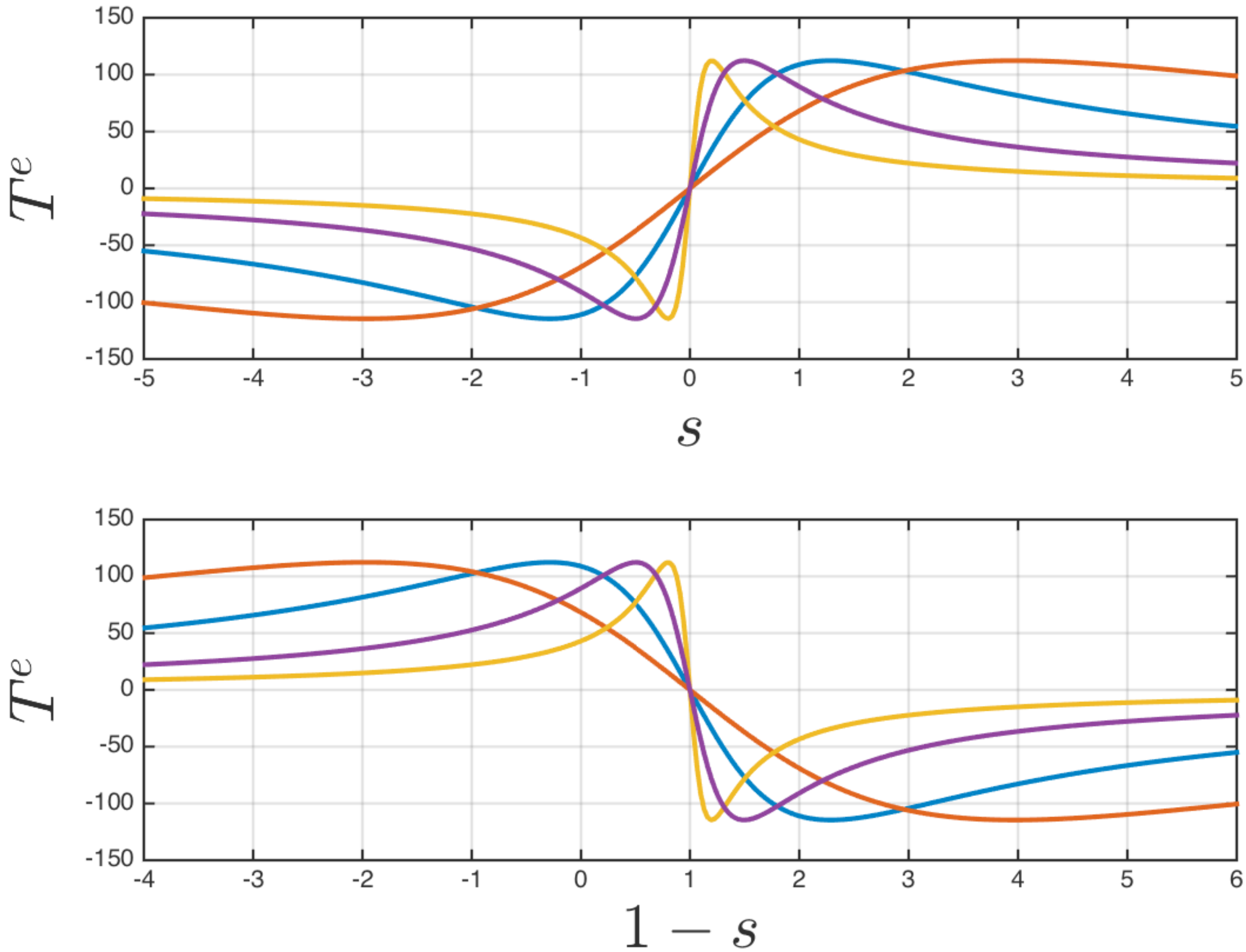
2. Often torque-slip characteristics is represented as torque-speed curve.



why $1-s$?

$\omega_m \propto 1-s$.

$\Rightarrow$ We have a torque speed curve.



150
100
50
0
-50
-100
-150
T^e
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-4
-3
-2
-1
0
1
2
3
4
5
s
150
100
50
0
-50
-100
-150
T^e
-4
-3
-2
-1
0
1
2
3
4
5
6
$1-s$